

SELECTING THE *SureStep*[™] STEPPING SYSTEM



APPENDIX

A

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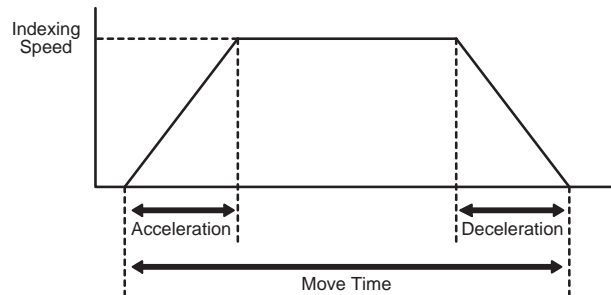
Selecting the *SureStep*™ Stepping System

The selection of your *SureStep*™ stepping system follows a defined process. Let's go through the process and define some useful relationships and equations. We will use this information to work some typical examples along the way.

The Selection Procedure

The motor provides for the required motion of the load through the actuator (mechanics that are between the motor shaft and the load or workpiece). Key information to accomplish the required motion is:

- total number of pulses from the PLC
- positioning resolution of the load
- indexing speed (or PLC pulse frequency) to achieve the move time
- required motor torque (including the 100% safety factor)
- load to motor inertia ratio



In the final analysis, we need to achieve the required motion with acceptable positioning accuracy.

How many pulses from the PLC to make the move?

The total number of pulses to make the entire move is expressed with the equation:

$$\text{Equation ①: } P_{\text{total}} = \text{total pulses} = (D_{\text{total}} \div (d_{\text{load}} \div i)) \times \theta_{\text{step}}$$

D_{total} = total move distance

d_{load} = lead or distance the load moves per revolution of the actuator's drive shaft
(P = pitch = $1/d_{\text{load}}$)

θ_{step} = driver step resolution (steps/rev_{motor})

i = gear reduction ratio (rev_{motor}/rev_{gearshaft})

Example 1: The motor is directly attached to a disk, the stepping driver is set at 400 steps per revolution and we need to move the disk 5.5 revolutions. How many pulses does the PLC need to send the driver?

$$\begin{aligned} P_{\text{total}} &= (5.5 \text{ rev}_{\text{disk}} \div (1 \text{ rev}_{\text{disk}}/\text{rev}_{\text{driveshaft}} \div 1 \text{ rev}_{\text{motor}}/\text{rev}_{\text{driveshaft}})) \\ &\quad \times 400 \text{ steps}/\text{rev}_{\text{motor}} \\ &= 2200 \text{ pulses} \end{aligned}$$

Example 2: The motor is directly attached to a ballscrew where one turn of the ballscrew results in 10 mm of linear motion, the stepping driver is set for 1000 steps per revolution, and we need to move 45 mm. How many pulses do we need to send the driver?

$$P_{\text{total}} = (45 \text{ mm} \div (10 \text{ mm/rev}_{\text{screw}} \div 1 \text{ rev}_{\text{motor}}/\text{rev}_{\text{screw}})) \times 1000 \text{ steps/rev}_{\text{motor}}$$

$$= 4500 \text{ pulses}$$

Example 3: Let's add a 2:1 belt reduction between the motor and ballscrew in example 2. Now how many pulses do we need to make the 45 mm move?

$$P_{\text{total}} = (45 \text{ mm} \div (10\text{mm/rev}_{\text{screw}} \div 2 \text{ rev}_{\text{motor}}/\text{rev}_{\text{screw}})) \times 1000 \text{ steps/rev}_{\text{motor}}$$

$$= 9000 \text{ pulses}$$

What is the positioning resolution of the load?

We want to know how far the load will move for one pulse or step of the motor shaft. The equation to determine the positioning resolution is:

Equation ②: L_{θ} = load positioning resolution = $(d_{\text{load}} \div i) \div \theta_{\text{step}}$

Example 4: What is the positioning resolution for the system in example 3?

$$L_{\theta} = (d_{\text{load}} \div i) \div \theta_{\text{step}}$$

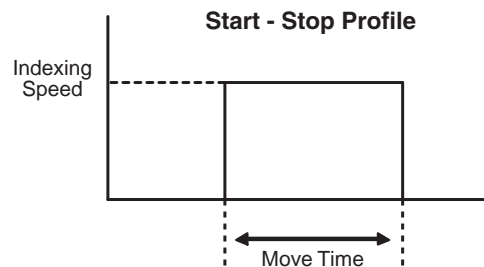
$$= (10 \text{ mm/rev}_{\text{screw}} \div 2 \text{ rev}_{\text{motor}}/\text{rev}_{\text{screw}}) \div 1000 \text{ steps/rev}_{\text{motor}}$$

$$= 0.005\text{mm/step}$$

$$\approx 0.0002"/\text{step}$$

What is the indexing speed to accomplish the move time?

The most basic type of motion profile is a "start-stop" profile where there is no acceleration or deceleration period. This type of motion profile is only used for low speed applications because the load is "jerked" from one speed to another and the stepping motor will stall or drop pulses if excessive speed changes are attempted. The equation to find indexing speed for "start-stop" motion is:



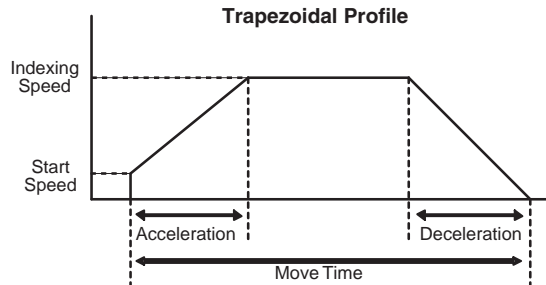
Equation ③: f_{SS} = indexing speed for start-stop profiles = $P_{\text{total}} \div t_{\text{total}}$

t_{total} = move time

Example 5: What is the indexing speed to make a "start-stop" move with 10,000 pulses in 800 ms?

$$f_{SS} = \text{indexing speed} = P_{\text{total}} \div t_{\text{total}} = 10,000 \text{ pulses} \div 0.8 \text{ seconds} \\ = 12,500 \text{ Hz.}$$

For higher speed operation, the "trapezoidal" motion profile includes controlled acceleration & deceleration and an initial non-zero starting speed. With the acceleration and deceleration periods equally set, the indexing speed can be found using the equation:



$$\text{Equation ④: } f_{\text{TRAP}} = (P_{\text{total}} - (f_{\text{start}} \times t_{\text{ramp}})) \div (t_{\text{total}} - t_{\text{ramp}}) \\ \text{for trapezoidal motion profiles}$$

f_{start} = starting speed

t_{ramp} = acceleration or deceleration time

Example 6: What is the required indexing speed to make a "trapezoidal" move in 800ms, accel/decel time of 200 ms each, 10,000 total pulses, and a starting speed of 40 Hz?

$$f_{\text{TRAP}} = (10,000 \text{ pulses} - (40 \text{ pulses/sec} \times 0.2 \text{ sec})) \div (0.8 \text{ sec} - 0.2 \text{ sec}) \\ \approx 16,653 \text{ Hz.}$$

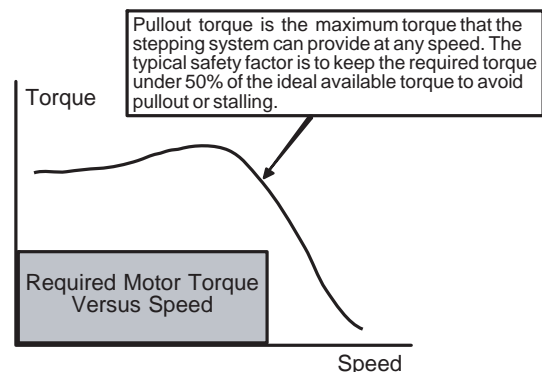
Calculating the Required Torque

The required torque from the stepping system is the sum of acceleration torque and the running torque. The equation for required motor torque is:

$$\text{Equation ⑤: } T_{\text{motor}} = T_{\text{accel}} + T_{\text{run}}$$

T_{accel} = motor torque required to accelerate and decelerate the total system inertia (including motor inertia)

T_{run} = constant motor torque requirement to run the mechanism due to friction, external load forces, etc.



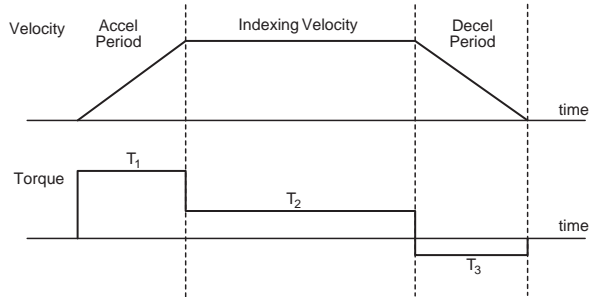
In **Table 1** we show how to calculate torque required to accelerate or decelerate an inertia from one speed to another and the calculation of running torque for common mechanical actuators.

Table 1 - Calculate the Torque for "Acceleration" and "Running"

The torque required to accelerate or decelerate an inertia with a linear change in velocity is:

Equation ⑥: $T_{\text{accel}} = J_{\text{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times (2\pi \div 60)$

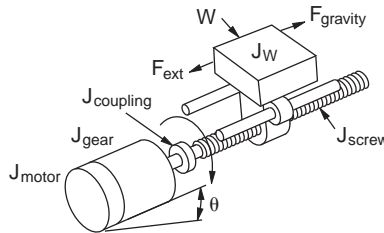
J_{total} is the motor inertia plus load inertia ("reflected" to the motor shaft). The $(2\pi \div 60)$ is a factor used to convert "change in speed" expressed in RPM into angular speed (radians/second). Refer to information in this table to calculate "reflected" load inertia for several common shapes and mechanical mechanisms.



Example 7: What is the required torque to accelerate an inertia of 0.002 lb-in-sec² (motor inertia is 0.0004 lb-in-sec² and "reflected" load inertia is 0.0016 lb-in-sec²) from zero to 600 RPM in 50 ms?

$T_{\text{accel}} = 0.002 \text{ lb-in-sec}^2 \times (600 \text{ RPM} \div 0.05 \text{ seconds}) \times (2\pi \div 60)$
 $\approx 2.5 \text{ lb-in}$

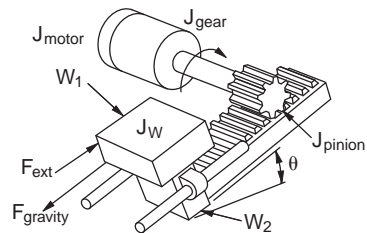
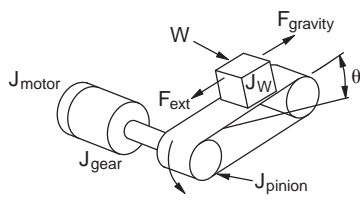
Leadscrew Equations



| Description: | Equations: |
|---|--|
| Motor RPM | $n_{\text{motor}} = (v_{\text{load}} \times P) \times i$, n_{motor} (RPM), v_{load} (in/min) |
| Torque required to accelerate and decelerate the load | $T_{\text{accel}} \approx J_{\text{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1$ |
| Motor total inertia | $J_{\text{total}} = J_{\text{motor}} + J_{\text{gear}} + ((J_{\text{coupling}} + J_{\text{screw}} + J_{\text{W}}) \div i^2)$ |
| Inertia of the load | $J_{\text{W}} = (W \div (g \times e)) \times (1 \div 2 \pi P)^2$ |
| Pitch and Efficiency | $P = \text{pitch} = \text{revs/inch of travel}$, $e = \text{efficiency}$ |
| Running torque | $T_{\text{run}} = ((F_{\text{total}} \div (2 \pi P)) + T_{\text{preload}}) \div i$ |
| Torque due to preload on the ballscrew | $T_{\text{preload}} = \text{ballscrew nut preload to minimize backlash}$ |
| Force total | $F_{\text{total}} = F_{\text{ext}} + F_{\text{friction}} + F_{\text{gravity}}$ |
| Force of gravity and Force of friction | $F_{\text{gravity}} = W \sin \theta$, $F_{\text{friction}} = \mu W \cos \theta$ |
| Incline angle and Coefficient of friction | $\theta = \text{incline angle}$, $\mu = \text{coefficient of friction}$ |

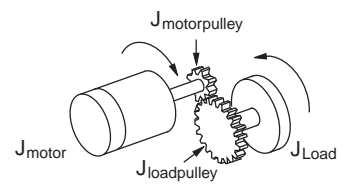
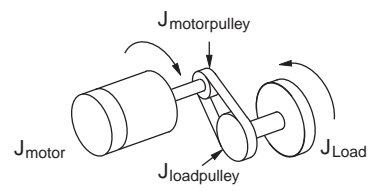
| Table 1 (cont'd) | | | |
|------------------------|----------------|-----------------------------|---------------------------|
| Typical Leadscrew Data | | | |
| Material: | e = efficiency | Material: | μ = coef. of friction |
| ball nut | 0.90 | steel on steel | 0.580 |
| acme with plastic nut | 0.65 | steel on steel (lubricated) | 0.150 |
| acme with metal nut | 0.40 | teflon on steel | 0.040 |
| | | ball bushing | 0.003 |

Belt Drive (or Rack & Pinion) Equations



| Description: | Equations: |
|---|---|
| Motor RPM | $n_{\text{motor}} = (v_{\text{load}} \times 2 \pi r) \times i$ |
| Torque required to accelerate and decelerate the load | $T_{\text{accel}} \approx J_{\text{total}} \times (\Delta \text{speed} \div \Delta \text{time}) \times 0.1$ |
| Inertia of the load | $J_{\text{total}} = J_{\text{motor}} + J_{\text{gear}} + ((J_{\text{pinion}} + J_{\text{W}}) \div i^2)$ |
| Inertia of the load | $J_{\text{W}} = (W \div (g \times e)) \times r^2$; $J_{\text{W}} = ((W_1 + W_2) \div (g \times e)) \times r^2$ |
| Radius of pulleys | $r = \text{radius of pinion or pulleys (inch)}$ |
| Running torque | $T_{\text{run}} = (F_{\text{total}} \times r) \div i$ |
| Force total | $F_{\text{total}} = F_{\text{ext}} + F_{\text{friction}} + F_{\text{gravity}}$ |
| Force of gravity and Force of friction | $F_{\text{gravity}} = W \sin \theta$; $F_{\text{friction}} = \mu W \cos \theta$ |

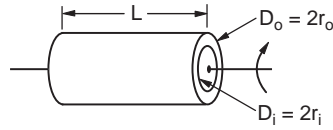
Belt (or Gear) Reducer Equations



| Description: | Equations: |
|---|---|
| Motor RPM | $n_{\text{motor}} = n_{\text{load}} \times i$ |
| Torque required to accelerate and decelerate the load | $T_{\text{accel}} \approx J_{\text{total}} \times (\Delta \text{speed} \div \Delta \text{time}) \times 0.1$ |
| Inertia of the load | $J_{\text{total}} = J_{\text{motor}} + J_{\text{motorpulley}} + ((J_{\text{loadpulley}} + J_{\text{Load}}) \div i^2)$ |
| Motor torque | $T_{\text{motor}} \times i = T_{\text{Load}}$ |

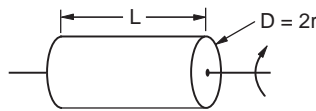
Table 1 (cont'd)

Inertia of Hollow Cylinder Equations



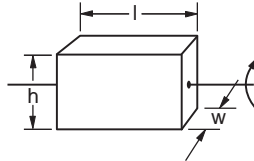
| Description: | Equations: |
|--------------|---|
| Inertia | $J = (W \times (r_o^2 + r_i^2)) \div (2g)$ |
| Inertia | $J = (\pi \times L \times \rho \times (r_o^4 - r_i^4)) \div (2g)$ |
| Volume | volume = $\pi/4 \times (D_o^2 - D_i^2) \times L$ |

Inertia of Solid Cylinder Equations



| Description: | Equations: |
|--------------|---|
| Inertia | $J = (W \times r^2) \div (2g)$ |
| Inertia | $J = (\pi \times L \times \rho \times r^4) \div (2g)$ |
| Volume | volume = $\pi \times r^2 \times L$ |

Inertia of Rectangular Block Equations



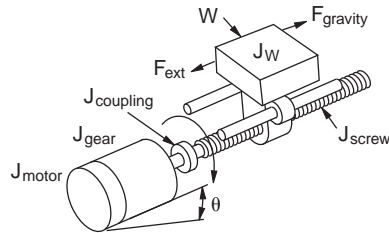
| Description: | Equations: |
|--------------|---------------------------------------|
| Inertia | $J = (W \div 12g) \times (h^2 + w^2)$ |
| Volume | volume = $l \times h \times w$ |

Symbol Definitions

| | |
|---------------------------------------|--|
| J = inertia | ρ = density |
| L = Length | ρ = 0.098 lb/in ³ (aluminum) |
| h = height | ρ = 0.28 lb/in ³ (steel) |
| w = width | ρ = 0.04 lb/in ³ (plastic) |
| W = weight | ρ = 0.31 lb/in ³ (brass) |
| D = diameter | ρ = 0.322 lb/in ³ (copper) |
| r = radius | |
| g = gravity = 386 in/sec ² | $\pi \approx 3.14$ |

Leadscrew - Example Calculations

Step 1 - Define the Actuator and Motion Requirements



- Weight of table and workpiece = 200 lb
- Angle of inclination = 0°
- Friction coefficient of sliding surfaces = 0.05
- External load force = 0
- Ball screw shaft diameter = 0.6 inch
- Ball screw length = 23.6 inch
- Ball screw material = steel
- Ball screw lead = 0.6 inch/rev ($P \approx 1.67$ rev/in)
- Desired Resolution = 0.001 inch/step
- Gear reducer = 2:1
- Stroke = 4.5 inch
- Move time = 1.7 seconds

| Definitions |
|---|
| d_{load} = lead or distance the load moves per revolution of the actuator's drive shaft ($P = \text{pitch} = 1/d_{load}$) |
| D_{total} = total move distance |
| θ_{step} = driver step resolution (steps/rev _{motor}) |
| i = gear reduction ratio (rev _{motor} /rev _{gearshaft}) |
| T_{accel} = motor torque required to accelerate and decelerate the total system inertia (including motor inertia) |
| T_{run} = constant motor torque requirement to run the mechanism due to friction, external load forces, etc. |
| t_{total} = move time |

Step 2 - Determine the Positioning Resolution of the Load

Rearranging Equation ④ to calculate the required stepping drive resolution:

$$\begin{aligned} \theta_{step} &= (d_{load} \div i) \div L_{\theta} \\ &= (0.6 \div 2) \div 0.001 \\ &= 300 \text{ steps/rev} \end{aligned}$$

With the 2:1 gear reduction, the stepping system can be set at 400 steps/rev to exceed the required load positioning resolution.

A 2:1 timing belt reducer is a good choice for low cost and low backlash. Also, the motor can be repositioned back under the leadscrew if desired with a timing belt reducer.

Step 3 - Determine the Motion Profile

From Equation ①, the total pulses to make the required move is:

$$P_{\text{total}} = (D_{\text{total}} \div (d_{\text{load}} \div i)) \times \theta_{\text{step}}$$

$$= (4.5 \div (0.6 \div 2)) \times 400 = 6,000 \text{ pulses}$$

From Equation ④, the indexing frequency for a trapezoidal move is:

$$f_{\text{TRAP}} = (P_{\text{total}} - (f_{\text{start}} \times t_{\text{ramp}})) \div (t_{\text{total}} - t_{\text{ramp}})$$

$$= (6,000 - (100 \times 0.43)) \div (1.7 - 0.43) \approx 4,690 \text{ Hz}$$

where accel time is 25% of total move time and starting speed is 100 Hz.

$$= 4,690 \text{ Hz} \times (60 \text{ sec}/1 \text{ min}) \div 400 \text{ steps/rev}$$

$$\approx 703 \text{ RPM motor speed}$$

Step 4 - Determine the Required Motor Torque

Using the equations in Table 1:

$$J_{\text{total}} = J_{\text{motor}} + J_{\text{gear}} + ((J_{\text{coupling}} + J_{\text{screw}} + J_{\text{W}}) \div i^2)$$

For this example, let's assume the gearbox and coupling inertia are zero.

$$J_{\text{W}} = (W \div (g \times e)) \times (1 \div 2 \times \pi P)^2$$

$$= (200 \div (386 \times 0.9)) \times (1 \div 2 \times 3.14 \times 1.67)^2$$

$$\approx 0.0052 \text{ lb-in-sec}^2$$

$$J_{\text{screw}} \approx (\pi \times L \times \rho \times r^4) \div (2g)$$

$$\approx (3.14 \times 23.6 \times 0.28 \times 0.3^4) \div (2 \times 386)$$

$$\approx 0.0002 \text{ lb-in-sec}^2$$

The inertia of the load and screw reflected to the motor is:

$$J_{\text{(screw + load) to motor}} = ((J_{\text{screw}} + J_{\text{W}}) \div i^2)$$

$$\approx ((0.0002 + 0.0052) \div 2^2) = 0.00135 \text{ lb-in-sec}^2$$

The torque required to accelerate the inertia is:

$$T_{\text{accel}} \approx J_{\text{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1$$

$$= 0.00135 \times (603 \div 0.2) \times 0.1 \approx 0.4 \text{ lb-in}$$

Next, we need to determine running torque. If the machine already exists then it is sometimes possible to actually measure running torque by turning the actuator driveshaft with a torque wrench.

$$T_{\text{run}} = ((F_{\text{total}} \div (2 \pi P)) + T_{\text{preload}}) \div i$$

$$F_{\text{total}} = F_{\text{ext}} + F_{\text{friction}} + F_{\text{gravity}}$$

$$= 0 + \mu W \cos\theta + 0 = 0.05 \times 200 = 10 \text{ lb}$$

$$T_{\text{run}} = (10 \div (2 \times 3.14 \times 1.66)) \div 2$$

$$\approx 0.48 \text{ lb-in}$$

where we have assumed preload torque to be zero.

From Equation ⑤, the required motor torque is:

$$T_{\text{motor}} = T_{\text{accel}} + T_{\text{run}} = 0.4 + 0.48 \approx 0.88 \text{ lb-in}$$

However, this is the required motor torque before we have picked a motor and included the motor inertia.

Step 5 - Select and Confirm the Stepping Motor and Driver System

It looks like a reasonable choice for a motor would be the STP-MTR-23055 or shorter NEMA 23. This motor has an inertia of:

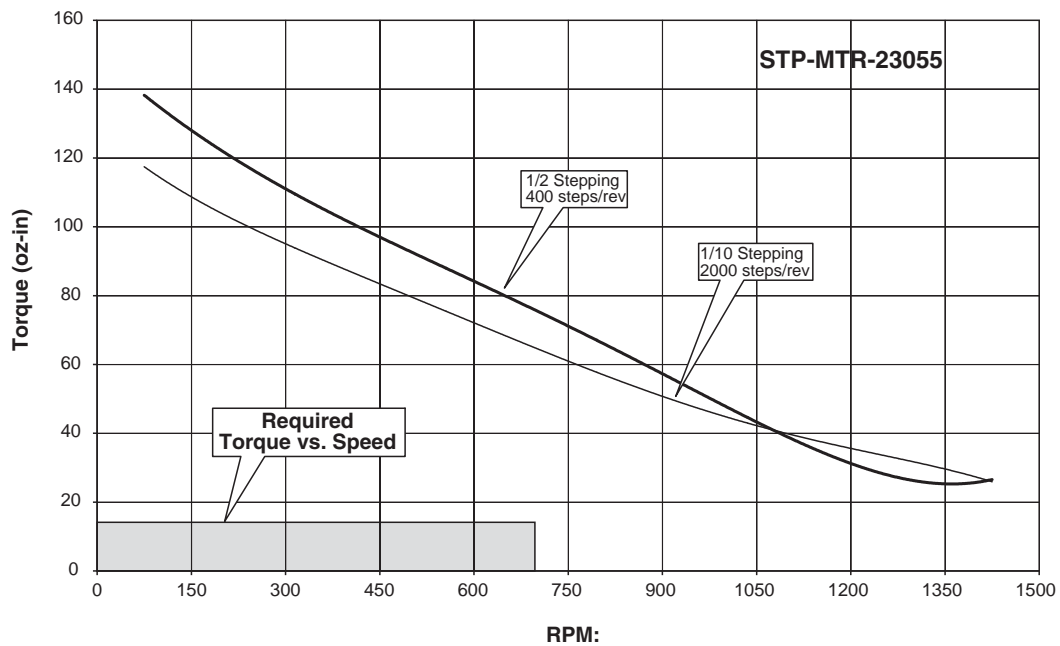
$$J_{\text{motor}} = 0.00024 \text{ lb-in-sec}^2$$

The actual motor torque would be modified:

$$\begin{aligned} T_{\text{accel}} &= J_{\text{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1 \\ &= (0.00135 + \mathbf{0.00024}) \times (603 \div 0.2) \times 0.1 \\ &\approx 0.48 \text{ lb-in} \end{aligned}$$

so that:

$$\begin{aligned} T_{\text{motor}} &= T_{\text{accel}} + T_{\text{run}} \\ &= 0.48 + 0.48 \approx 0.96 \text{ lb-in} \approx 16 \text{ oz-in} \end{aligned}$$



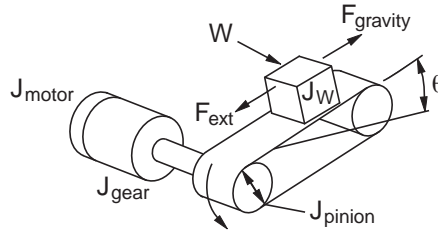
It looks like the STP-MTR-23055 stepping motor will work. However, we still need to check the load to motor inertia ratio:

$$\begin{aligned} \text{Ratio} &= J_{\text{(screw + load) to motor}} \div J_{\text{motor}} \\ &= 0.00135 \div 0.00024 = 5.625 \end{aligned}$$

It is best to keep the load to motor inertia ratio below 10 so 5.625 is within an acceptable range. For additional comfort, you could move up to the STP-MTR-23079 or the larger NEMA 23 motor. In this case, the load to motor inertia ratio would be lowered to 3.2.

Belt Drive - Example Calculations

Step 1 - Define the Actuator and Motion Requirements



- Weight of table and workpiece = 3 lb
- External force = 0 lb
- Friction coefficient of sliding surfaces = 0.05
- Angle of table = 0°
- Belt and pulley efficiency = 0.8
- Pulley diameter = 1.5 inch
- Pulley thickness = 0.75 inch
- Pulley material = aluminum
- Desired Resolution = 0.001 inch/step
- Gear Reducer = 5:1
- Stroke = 50 inch
- Move time = 4.0 seconds
- Accel and decel time = 1.0 seconds

| Definitions |
|---|
| d_{load} = lead or distance the load moves per revolution of the actuator's drive shaft ($P = \text{pitch} = 1/d_{load}$) |
| D_{total} = total move distance |
| θ_{step} = driver step resolution (steps/rev _{motor}) |
| i = gear reduction ratio (rev _{motor} /rev _{gearshaft}) |
| T_{accel} = motor torque required to accelerate and decelerate the total system inertia (including motor inertia) |
| T_{run} = constant motor torque requirement to run the mechanism due to friction, external load forces, etc. |
| t_{total} = move time |

Step 2 - Determine the Positioning Resolution of the Load

Rearranging Equation ④ to calculate the required stepping drive resolution:

$$\begin{aligned} \theta_{step} &= (d_{load} \div i) \div L_{\theta} \\ &= ((3.14 \times 1.5) \div 5) \div 0.001 \\ &= 942 \text{ steps/rev} \\ &\text{where } d_{load} = \pi \times \text{Pulley Diameter.} \end{aligned}$$

With the 5:1 gear reduction, the stepping system can be set at 1000 steps/rev to slightly exceed the required load positioning resolution.

Reduction is almost always required with a belt drive and a 5:1 planetary gearhead is common.

Step 3 - Determine the Motion Profile

From **Equation ①**, the total pulses to make the required move is:

$$\begin{aligned} P_{\text{total}} &= (D_{\text{total}} \div (d_{\text{load}} \div i)) \times \theta_{\text{step}} \\ &= 50 \div ((3.14 \times 1.5) \div 5) \times 1000 \\ &\approx 53,079 \text{ pulses} \end{aligned}$$

From **Equation ④**, the running frequency for a trapezoidal move is:

$$\begin{aligned} f_{\text{TRAP}} &= (P_{\text{total}} - (f_{\text{start}} \times t_{\text{ramp}})) \div (t_{\text{total}} - t_{\text{ramp}}) \\ &= 53,079 \div (4 - 1) \\ &\approx 17,693 \text{ Hz} \end{aligned}$$

where accel time is 25% of total move time and starting speed is zero.

$$\begin{aligned} &= 17,693 \text{ Hz} \times (60 \text{ sec}/1 \text{ min}) \div 1000 \text{ steps/rev} \\ &\approx 1,062 \text{ RPM motor speed} \end{aligned}$$

Step 4 - Determine the Required Motor Torque

Using the equations in **Table 1**:

$$J_{\text{total}} = J_{\text{motor}} + J_{\text{gear}} + ((J_{\text{pulleys}} + J_{\text{W}}) \div i^2)$$

For this example, let's assume the gearbox inertia is zero.

$$\begin{aligned} J_{\text{W}} &= (W \div (g \times e)) \times r^2 \\ &= (3 \div (386 \times 0.8)) \times 0.752 \\ &\approx 0.0055 \text{ lb-in-sec}^2 \end{aligned}$$

Pulley inertia (remember there are two pulleys) can be calculated as:

$$\begin{aligned} J_{\text{pulleys}} &\approx ((\pi \times L \times \rho \times r^4) \div (2g)) \times 2 \\ &\approx ((3.14 \times 0.75 \times 0.098 \times 0.754) \div (2 \times 386)) \times 2 \\ &\approx 0.00019 \text{ lb-in-sec}^2 \end{aligned}$$

The inertia of the load and pulleys reflected to the motor is:

$$\begin{aligned} J_{(\text{pulleys} + \text{load}) \text{ to motor}} &= ((J_{\text{pulleys}} + J_{\text{W}}) \div i^2) \\ &\approx ((0.0055 + 0.00019) \div 52) \approx 0.00023 \text{ lb-in-sec}^2 \end{aligned}$$

The torque required to accelerate the inertia is:

$$\begin{aligned} T_{\text{acc}} &\approx J_{\text{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1 \\ &= 0.00023 \times (1062 \div 1) \times 0.1 \\ &= 0.025 \text{ lb-in} \end{aligned}$$

$$T_{\text{run}} = (F_{\text{total}} \times r) \div i$$

$$\begin{aligned} F_{\text{total}} &= F_{\text{ext}} + F_{\text{friction}} + F_{\text{gravity}} \\ &= 0 + \mu W \cos\theta + 0 = 0.05 \times 3 = 0.15 \text{ lb} \end{aligned}$$

$$\begin{aligned} T_{\text{run}} &= (0.15 \times 0.75) \div 5 \\ &\approx 0.0225 \text{ lb-in} \end{aligned}$$

From **Equation ⑤**, the required motor torque is:

$$T_{\text{motor}} = T_{\text{accel}} + T_{\text{run}} = 0.025 + 0.0225 \approx 0.05 \text{ lb-in}$$

However, this is the required motor torque before we have picked a motor and included the motor inertia.

Step 5 - Select and Confirm the Stepping Motor and Driver System

It looks like a reasonable choice for a motor would be the STP-MTR-17048 or NEMA 17 motor. This motor has an inertia of:

$$J_{\text{motor}} = 0.00006 \text{ lb-in-sec}^2$$

The actual motor torque would be modified:

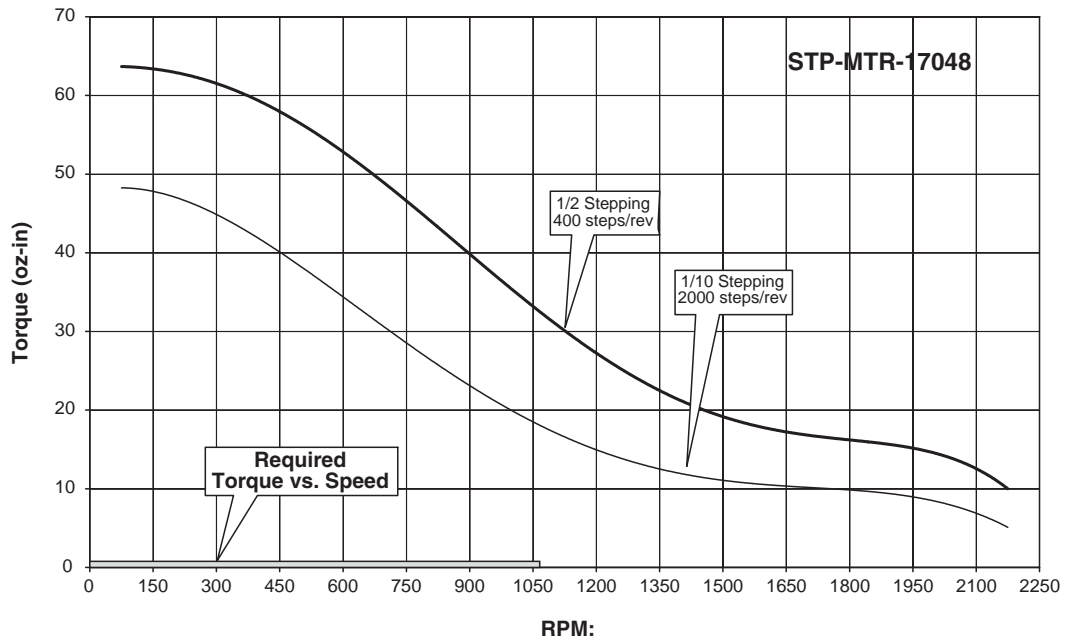
$$T_{\text{accel}} = J_{\text{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1$$

$$= (0.00023 + 0.00006) \times (1062 \div 1) \times 0.1 \approx 0.03 \text{ lb-in}$$

so that:

$$T_{\text{motor}} = T_{\text{accel}} + T_{\text{run}}$$

$$= 0.03 + 0.0225 \approx 0.0525 \text{ lb-in} \approx 0.84 \text{ oz-in}$$



It looks like the STP-MTR-17048 stepping motor will work. However, we still need to check the load to motor inertia ratio:

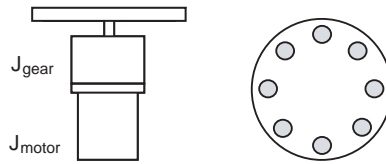
$$\text{Ratio} = J_{(\text{pulleys} + \text{load}) \text{ to motor}} \div J_{\text{motor}}$$

$$= 0.00023 \div 0.00006 = 3.8$$

It is best to keep the load to motor inertia ratio below 10 so 3.8 is within an acceptable range.

Index Table - Example Calculations

Step 1 - Define the Actuator and Motion Requirements



Diameter of index table = 12 inch
 Thickness of index table = 2 inch
 Table material = steel
 Number of workpieces = 8
 Desired Resolution = 0.036°
 Gear Reducer = 25:1
 Index angle = 45°
 Index time = 0.7 seconds

| Definitions |
|---|
| d_{load} = lead or distance the load moves per revolution of the actuator's drive shaft ($P = \text{pitch} = 1/d_{load}$) |
| D_{total} = total move distance |
| θ_{step} = driver step resolution (steps/rev _{motor}) |
| i = gear reduction ratio (rev _{motor} /rev _{gearshaft}) |
| T_{accel} = motor torque required to accelerate and decelerate the total system inertia (including motor inertia) |
| T_{run} = constant motor torque requirement to run the mechanism due to friction, external load forces, etc. |
| t_{total} = move time |

Step 2 - Determine the Positioning Resolution of the Load

Rearranging **Equation ④** to calculate the required stepping drive resolution:

$$\begin{aligned} \theta_{step} &= (d_{load} \div i) \div L_{\theta} \\ &= (360^\circ \div 25) \div 0.036^\circ \\ &= 400 \text{ steps/rev} \end{aligned}$$

With the 25:1 gear reduction, the stepping system can be set at 400 steps/rev to equal the required load positioning resolution.

It is almost always necessary to use significant gear reduction when controlling a large inertia disk.

Step 3 - Determine the Motion Profile

From Equation ①, the total pulses to make the required move is:

$$\begin{aligned} P_{\text{total}} &= (D_{\text{total}} \div (d_{\text{load}} \div i)) \times \theta_{\text{step}} \\ &= (45^\circ \div (360^\circ \div 25)) \times 400 \\ &= 1250 \text{ pulses} \end{aligned}$$

From Equation ④, the running frequency for a trapezoidal move is:

$$\begin{aligned} f_{\text{TRAP}} &= (P_{\text{total}} - (f_{\text{start}} \times t_{\text{ramp}})) \div (t_{\text{total}} - t_{\text{ramp}}) \\ &= 1,250 \div (0.7 - 0.17) \approx 2,360 \text{ Hz} \\ &\text{where accel time is 25\% of total move time and starting speed is zero.} \\ &= 2,360 \text{ Hz} \times (60 \text{ sec}/1 \text{ min}) \div 400 \text{ steps/rev} \\ &\approx 354 \text{ RPM} \end{aligned}$$

Step 4 - Determine the Required Motor Torque

Using the equations in Table 1:

$$J_{\text{total}} = J_{\text{motor}} + J_{\text{gear}} + (J_{\text{table}} \div i^2)$$

For this example, let's assume the gearbox inertia is zero.

$$\begin{aligned} J_{\text{table}} &\approx (\pi \times L \times \rho \times r^4) \div (2g) \\ &\approx (3.14 \times 2 \times 0.28 \times 1296) \div (2 \times 386) \\ &\approx 2.95 \text{ lb-in-sec}^2 \end{aligned}$$

The inertia of the indexing table reflected to the motor is:

$$\begin{aligned} J_{\text{table to motor}} &= J_{\text{table}} \div i^2 \\ &\approx 0.0047 \text{ lb-in-sec}^2 \end{aligned}$$

The torque required to accelerate the inertia is:

$$\begin{aligned} T_{\text{accel}} &\approx J_{\text{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1 \\ &= 0.0047 \times (354 \div 0.17) \times 0.1 \\ &\approx 1.0 \text{ lb-in} \end{aligned}$$

From Equation ⑤, the required motor torque is:

$$\begin{aligned} T_{\text{motor}} &= T_{\text{accel}} + T_{\text{run}} \\ &= 1.0 + 0 = 1.0 \text{ lb-in} \end{aligned}$$

However, this is the required motor torque before we have picked a motor and included the motor inertia.

Step 5 - Select and Confirm the Stepping Motor and Driver System

It looks like a reasonable choice for a motor would be the STP-MTR-34066 or NEMA 34 motor. This motor has an inertia of:

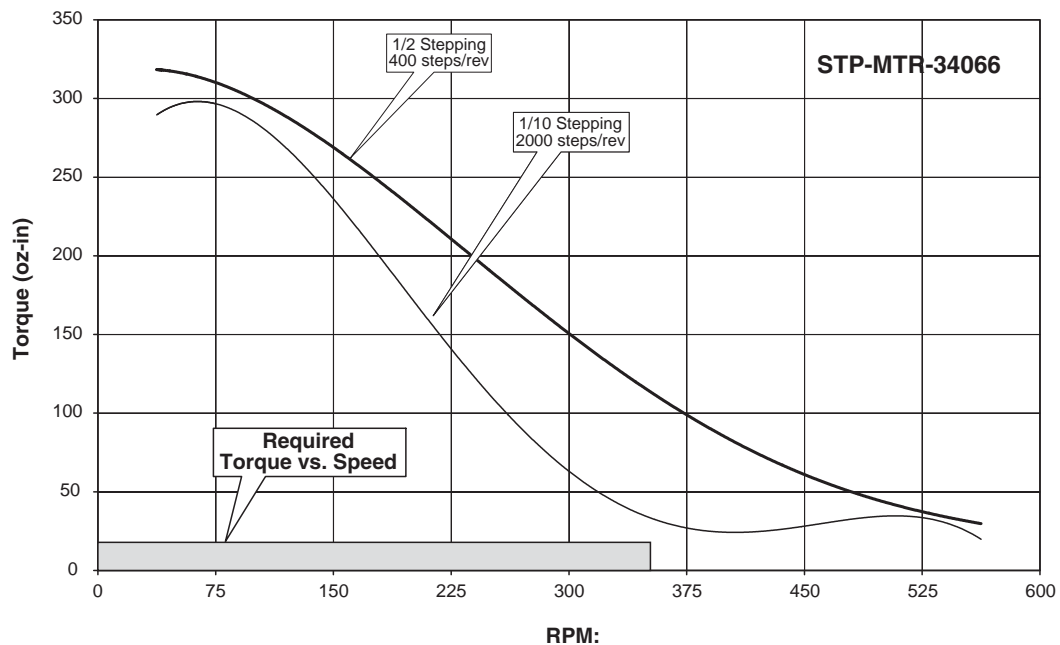
$$J_{\text{motor}} = 0.0012 \text{ lb-in-sec}^2$$

The actual motor torque would be modified:

$$\begin{aligned} T_{\text{accel}} &= J_{\text{total}} \times (\Delta\text{speed} \div \Delta\text{time}) \times 0.1 \\ &= (0.0047 + \mathbf{0.0012}) \times (354 \div 0.17) \times 0.1 \\ &\approx 1.22 \text{ lb-in} \end{aligned}$$

so that:

$$\begin{aligned} T_{\text{motor}} &= T_{\text{accel}} + T_{\text{run}} \\ &= 1.22 + 0 \\ &= 1.22 \text{ lb-in} = 19.52 \text{ oz-in} \end{aligned}$$



It looks like the STP-MTR-34066 stepping motor will work. However, we still need to check the load to motor inertia ratio:

$$\begin{aligned} \text{Ratio} &= J_{\text{table to motor}} \div J_{\text{motor}} \\ &= 0.0047 \div 0.0012 = 3.9 \end{aligned}$$

It is best to keep the load to motor inertia ratio below 10 so 3.9 is within an acceptable range.

Engineering Unit Conversion Tables, Formulae, & Definitions:

| Conversion of Length | | | | | | | |
|--|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| To convert A to B, multiply A by the entry in the table. | | B | | | | | |
| | | µm | mm | m | mil | in | ft |
| A | µm | 1 | 1.000E-03 | 1.000E-06 | 3.937E-02 | 3.937E-05 | 3.281E-06 |
| | mm | 1.000E+03 | 1 | 1.000E-03 | 3.937E+01 | 3.937E-02 | 3.281E-03 |
| | m | 1.000E+06 | 1.000E+03 | 1 | 3.937E+04 | 3.937E+01 | 3.281E+00 |
| | mil | 2.540E+01 | 2.540E-02 | 2.540E-05 | 1 | 1.000E-03 | 8.330E-05 |
| | in | 2.540E+04 | 2.540E+01 | 2.540E-02 | 1.000E+03 | 1 | 8.330E-02 |
| | ft | 3.048E+05 | 3.048E+02 | 3.048E-01 | 1.200E+04 | 1.200E+01 | 1 |

| Conversion of Torque | | | | | | | |
|--|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| To convert A to B, multiply A by the entry in the table. | | B | | | | | |
| | | Nm | kpm(kg-m) | kg-cm | oz-in | lb-in | lb-ft |
| A | Nm | 1 | 1.020E-01 | 1.020E+01 | 1.416E+02 | 8.850E+00 | 7.380E-01 |
| | kpm(kg-m) | 9.810E+00 | 1 | 1.000E+02 | 1.390E+03 | 8.680E+01 | 7.230E+00 |
| | kg-cm | 9.810E-02 | 1.000E-02 | 1 | 1.390E+01 | 8.680E-01 | 7.230E-02 |
| | oz-in | 7.060E-03 | 7.200E-04 | 7.200E-02 | 1 | 6.250E-02 | 5.200E-03 |
| | lb-in | 1.130E-01 | 1.150E-02 | 1.150E+00 | 1.600E+01 | 1 | 8.330E-02 |
| | lb-ft | 1.356E+00 | 1.380E-01 | 1.383E+01 | 1.920E+02 | 1.200E+01 | 1 |

| Conversion of Moment of Inertia | | | | | | | | |
|--|----------------------|-------------------|----------------------|----------------------|----------------------|--------------------|--------------------|--------------------|
| To convert A to B, multiply A by the entry in the table. | | B | | | | | | |
| | | kg-m ² | kg-cm-s ² | oz-in-s ² | lb-in-s ² | oz-in ² | lb-in ² | lb-ft ² |
| A | kg-m ² | 1 | 1.020E+01 | 1.416E+02 | 8.850E+00 | 5.470E+04 | 3.420E+03 | 2.373E+01 |
| | kg-cm-s ² | 9.800E-02 | 1 | 1.388E+01 | 8.680E-01 | 5.360E+03 | 3.350E+02 | 2.320E+00 |
| | oz-in-s ² | 7.060E-03 | 7.190E-02 | 1 | 6.250E-02 | 3.861E+02 | 2.413E+01 | 1.676E-01 |
| | lb-in-s ² | 1.130E-01 | 1.152E+00 | 1.600E+01 | 1 | 6.180E+03 | 3.861E+02 | 2.681E+00 |
| | oz-in ² | 1.830E-05 | 1.870E-04 | 2.590E-03 | 1.620E-04 | 1 | 6.250E-02 | 4.340E-04 |
| | lb-in ² | 2.930E-04 | 2.985E-03 | 4.140E-02 | 2.590E-03 | 1.600E+01 | 1 | 6.940E-03 |
| | lb-ft ² | 4.210E-02 | 4.290E-01 | 5.968E+00 | 3.730E-01 | 2.304E+03 | 1.440E+02 | 1 |

Engineering Unit Conversion Tables, Formulae, & Definitions (cont'd):

| General Formulae & Definitions | |
|--------------------------------|--|
| Description: | Equations: |
| Gravity | gravity = 9.8 m/s ² ; 386 in/s ² |
| Torque | $T = J \cdot \alpha$; $\alpha = \text{rad/s}^2$ |
| Power (Watts) | $P (W) = T (N \cdot m) \cdot \omega (\text{rad/s})$ |
| Power (Horsepower) | $P (hp) = T (\text{lb} \cdot \text{in}) \cdot \nu (\text{rpm}) / 63,024$ |
| Horsepower | 1 hp = 746W |
| Revolutions | 1 rev = 1,296,000 arc-sec / 21,600 arc-min |

| Equations for Straight-Line Velocity & Constant Acceleration | |
|--|---|
| Description: | Equations: |
| Final velocity | $v_f = v_i + at$ final velocity = (initial velocity) + (acceleration)(time) |
| Final position | $x_f = x_i + \frac{1}{2}(v_i + v_f)t$ final position = initial position + [(1/2)(initial velocity + final velocity)(time)] |
| Final position | $x_f = x_i + v_i t + \frac{1}{2}at^2$ final position = initial position + (initial velocity)(time) + (1/2)(acceleration)(time squared) |
| Final velocity squared | $v_f^2 = v_i^2 + 2a(x_f - x_i)$ final velocity squared = initial velocity squared + [(2)(acceleration)(final position - initial position)] |